

# SERIES CONVERGENCE/DIVERGENCE FLOW CHART

## TEST FOR DIVERGENCE

Does  $\lim_{n \rightarrow \infty} a_n = 0$ ?

NO

$\sum a_n$  Diverges

YES

## p-SERIES

Does  $a_n = 1/n^p, n \geq 1$ ?

YES

Is  $p > 1$ ?

YES

$\sum a_n$  Converges

NO

$\sum a_n$  Diverges

NO

## GEOMETRIC SERIES

Does  $a_n = ar^{n-1}, n \geq 1$ ?

YES

Is  $|r| < 1$ ?

YES

$\sum_{n=1}^{\infty} a_n = \frac{a}{1-r}$

NO

$\sum a_n$  Diverges

NO

## ALTERNATING SERIES

Does  $a_n = (-1)^n b_n$  or  $a_n = (-1)^{n-1} b_n, b_n \geq 0$ ?

YES

Is  $b_{n+1} \leq b_n$  &  $\lim_{n \rightarrow \infty} b_n = 0$ ?

YES

$\sum a_n$  Converges

NO

## TELESCOPING SERIES

Do subsequent terms cancel out previous terms in the sum? May have to use partial fractions, properties of logarithms, etc. to put into appropriate form.

YES

Does  $\lim_{n \rightarrow \infty} s_n = s$  finite?

YES

$\sum a_n = s$

NO

$\sum a_n$  Diverges

NO

## TAYLOR SERIES

Does  $a_n = \frac{f^{(n)}(a)}{n!} (x-a)^n$ ?

YES

Is  $x$  in interval of convergence?

YES

$\sum_{n=0}^{\infty} a_n = f(x)$

NO

$\sum a_n$  Diverges

NO

Try one or more of the following tests:

## COMPARISON TEST

Pick  $\{b_n\}$ . Does  $\sum b_n$  converge?

YES

Is  $0 \leq a_n \leq b_n$ ?

YES

$\sum a_n$  Converges

NO

Is  $0 \leq b_n \leq a_n$ ?

YES

$\sum a_n$  Diverges

NO

## LIMIT COMPARISON TEST

Pick  $\{b_n\}$ . Does  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$  finite &  $a_n, b_n > 0$ ?

YES

Does  $\sum_{n=1}^{\infty} b_n$  converge?

YES

$\sum a_n$  Converges

NO

$\sum a_n$  Diverges

## INTEGRAL TEST

Does  $a_n = f(n), f(x)$  is continuous, positive & decreasing on  $[a, \infty)$ ?

YES

Does  $\int_a^{\infty} f(x) dx$  converge?

YES

$\sum_{n=a}^{\infty} a_n$  Converges

NO

$\sum a_n$  Diverges

## RATIO TEST

Is  $\lim_{n \rightarrow \infty} |a_{n+1}/a_n| \neq 1$ ?

YES

Is  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$ ?

YES

$\sum a_n$  Abs. Conv.

NO

$\sum a_n$  Diverges

## ROOT TEST

Is  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} \neq 1$ ?

YES

Is  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} < 1$ ?

YES

$\sum a_n$  Abs. Conv.

NO

$\sum a_n$  Diverges